Kalman Filter Analysis

**CUNY SPS MSDS**

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In this analysis, we use the Kalman Filter to estimate the values of the assets in our investment portfolio.

We make the following assumptions in our analysis:

* The asset value follows a Brownian motion. That is, the daily change of the value is normally distributed with expected value 0.
* People's estimation of the true asset value is normally distributed with expected value approximately the same as the true asset value.
* The true asset value doesn't change through the same day. So the trading prices on the same day are the estimations of the same asset value by different traders.

Based on the assumptions, we have the following Predictor Equation and Measurement Equation:

where is normally distributed with variance Q.

where is normally distributed with variance R.

We define the terms in the Kalman Filter as following:

* State (**x**): The true asset value. This is not the trading price of the asset. In real time transactions, people are estimating the value of the same asset differently and hence the trading price may differ from the true asset value.
* Measurement (**z**): We will use the daily VWAP (volume weighted average price) as the measurement or observation instead of the daily closing price. The daily closing price is just the trading price of the last transaction of the day and so it doesn't reflect the true state of the asset.
* Process Noise Uncertainty (**Q**): The random effect on the change of the state (asset value). Assume that the asset value follows a Brownian motion. The change of the asset value is normally distributed with expected value 0 with a variance Q.
* Measurement Uncertainty (**R**): The variance of the measurement error. In this case, the error is the amount of mispricing of the true asset value, which is *trading price - true asset value*. We assume this error is normally distributed with expected value 0 and variance R.

Since we use the VWAP as the measurement of the asset value, we can estimate Q using the volume weighted variance of (typical price – VWAP), where the typical price = (1/3)\*(high + low + close) as it is calculated in the VWAP calculation. For R, we can estimate it using the variance of the percent change of the daily VWAP.

Usually, Q and R are constant in the Kalman Filter calculation, but we would like the values of Q and R to be dynamic, which change based on the predicted value in each iteration. It is reasonable that a person misprices an asset by 10 when the price is 200. However, it is unreasonable that a person misprices an asset by 10 when the price is 20. Therefore, we construct our own Kalman Filter calculation and update Q and R in each iteration.

In this analysis, we demonstrate the Kalman Filter estimation for 'ARLP', one of our assets in our investment portfolio. The first and last few iterations of our calculation is showed below.

|  | **VWAP** | **x\_predict** | **P\_predict** | **innovation** | **R** | **Q** | **P** | **K** | **KF\_Value** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **2022-07-29** | 21.950682 | 23.828603 | 6.38588 | -1.877921 | 0.088585 | 0.39084 | 0.087373 | 0.986318 | 21.976376 |
| **2022-08-01** | 22.032092 | 21.976376 | 0.419814 | 0.055715 | 0.075349 | 0.33244 | 0.063883 | 0.84783 | 22.023613 |
| **2022-08-02** | 22.549370 | 22.023613 | 0.397754 | 0.525757 | 0.075673 | 0.333871 | 0.063578 | 0.840159 | 22.465333 |
| **2022-08-03** | 23.023072 | 22.465333 | 0.410975 | 0.557739 | 0.078739 | 0.347398 | 0.066079 | 0.839214 | 22.933395 |
| **2022-08-04** | 22.420136 | 22.933395 | 0.428104 | -0.513259 | 0.082054 | 0.362025 | 0.068857 | 0.839159 | 22.502689 |

|  | **VWAP** | **x\_predict** | **P\_predict** | **innovation** | **R** | **Q** | **P** | **K** | **KF\_Value** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **2022-10-17** | 22.763507 | 22.499246 | 0.418301 | 0.264261 | 0.078977 | 0.348447 | 0.066434 | 0.841181 | 22.721537 |
| **2022-10-18** | 22.937638 | 22.721537 | 0.421801 | 0.2161 | 0.080545 | 0.355367 | 0.067631 | 0.839662 | 22.902988 |
| **2022-10-19** | 22.779246 | 22.902988 | 0.428696 | -0.123742 | 0.081837 | 0.361065 | 0.068719 | 0.839703 | 22.799082 |
| **2022-10-20** | 22.898098 | 22.799082 | 0.426515 | 0.099016 | 0.081096 | 0.357796 | 0.06814 | 0.84024 | 22.882279 |
| **2022-10-21** | 22.734939 | 22.882279 | 0.428553 | -0.14734 | 0.081689 | 0.360413 | 0.068611 | 0.839901 | 22.758528 |

We can see that the Kalman gain quickly converge to around 0.84. The high Kalman gain indicates that innovation makes the most contribution to the estimated value and the predicted value from the last period only plays a small role. This is consistent with the fact that the asset value is difficult to be predicted.

We can confirm from the following graph that the Kalman Filter estimations are mostly following the actual VWAP values.

Chart, line chart

Description automatically generated

To verify that our calculation is reasonable, we can check the percentage of the innovations that are within 1.96 standard deviation of the prediction. The result is 0.9167. The number is close to 90%, which is consistent with our normal distribution assumption. Our calculation is reasonable.

Since we assume that the asset value following a Brownian motion, we predict the asset value of the following four days to be the same as the last Kalman Filter estimation. Let's compare the actual VWAP of the four days and our prediction.

|  | **Actual VWAP** | **Prediction** | **Prediction Std** | **Error** | **Number of Std** |
| --- | --- | --- | --- | --- | --- |
| **2022-10-24** | 22.651619 | 22.758528 | 0.652024 | -0.106909 | -0.163965 |
| **2022-10-25** | 23.311725 | 22.758528 | 0.884116 | 0.553197 | 0.625706 |
| **2022-10-26** | 23.588834 | 22.758528 | 1.066858 | 0.830306 | 0.778272 |
| **2022-10-27** | 24.052763 | 22.758528 | 1.222583 | 1.294235 | 1.058607 |

We calculated the predicted variance increasingly by adding the Process Noise Uncertainty variance for each day. The result shows that the actual VWAP is within 1.1 standard deviation of our prediction. Our Kalman Filter calculation gives plausible predictions and can be used to manage our portfolio volatility / risk.